TOPIC FOR FRIDAY NOVEMBER 13^{th}

PYTHAGOREAN TRIPLES AND THEIR GENERALIZATIONS

The *Pythagorean Theorem* says that if a right triangle has legs of length x and y, and hypotenuse of length z, then x, y, z must satisfy the equality

$$x^2 + y^2 = z^2$$

There are many ways to see that this is true, and you probably went over at least one of these ways in a geometry class at some point. How should we go about finding solutions to this equation? That is, how do we find x, y, z making the equation true? First we have to decide what types of solutions we will consider. For instance, if we let x, y, z be any *real numbers*, then we get *uncountably* many solutions corresponding to points on circles (can you see why?). But what if we only consider x, y, z that have integer values?

A *Pythagorean triple* is a triple (x, y, z) satisfying the above equality with x, y, z all integers. The most common example of a Pythagorean triple is $(3, 4, 5)$. Indeed we see that

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

One can check that $(5, 12, 13)$ and $(7, 24, 25)$ are also Pythagorean triples and think of ways to find many more. The main focus of this meeting will be to explain how to find **all** Pythagorean triples and to explicitly write out a way to find them.

Depending on how this goes, we will also talk about how this problem fits in a larger family of problems. Notice that if we divide both sides of the equation by z^2 we get that

$$x^2 + y^2 = z^2 \iff \left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2 = 1$$

which (after a little work with common denominators) tells us that Pythagorean triples correspond to so called *rational points on the unit circle*. A circle is an example of a *conic section*. It can be shown that every degree 2 polynomial in two variables x and y can be described by a conic section. Thus we can solve every such polynomial with integer solutions if and only if we can find the corresponding rational points on the appropriate conic section.

In the mathematical sciences, the usefullness of solving polynomial equations cannot be overstated. A modern example of difficult equation solving is the *human genome project*. While techniques exist for solving such equations over the complex and real numbers, finding rational and integral solutions remains a much more difficult task. This meeting will be an introduction to the mathematical theory behind such problems.